The KSBA compactification of the moduli space of $D_{1,6}$-polarized Enriques surfaces

Luca Schaffler, University of Georgia
JMM 2017, Atlanta
AMS Contributed Paper Session on Algebraic Geometry

January 6, 2017

Motivation

Important problem: given a moduli space M, provide a compactification $\overline{M}$ with "good" properties:

- Functorial;
- Natural;
- Interesting structure to study.

Examples

"Model" compactifications: $\overline{M}_{g,n}$, $\overline{A}_g^{2^{\mathrm{nd}}\,Vor.}$.

My focus is on **Enriques surfaces**.

Compactification via **stable pairs**: (X, B) semi log canonical with $K_X + B$ ample (KSBA compactification).

$D_{1,6}$-polarized Enriques surfaces

- **Enriques surface** S**:** smooth projective connected 2-dim. variety$/\mathbb{C}$ such that $2K_S \sim 0$ and $h^0(S, \omega_S), h^1(S, \mathcal{O}_S) = 0$.
- S is $D_{1,6}$-**polarized** if $D_{1,6} \overset{\text{primitive}}{\hookrightarrow} \mathrm{Pic}(S)$ with extra conditions (this describes a 4-dimensional family).
- $D_{1,6} \subset \mathbb{Z} \oplus \mathbb{Z}^6(-1)$ sublattice of vectors of even square.

Why this example?

The choice of divisor and the moduli space of stable pairs

Let S be a $D_{1,6}$-polarized Enriques surface.

- $D_{1,6}$-polarization on S $\implies$ three genus 1 fibrations on S $\implies$ six half-fibers $E_1, E_1', E_2, E_2', E_3, E_3'$.
- Consider stable pairs $(S, \epsilon\sum_{i=1}^{3}(E_i + E_i'))$, $0 < \epsilon \ll 1$, $\epsilon \in \mathbb{Q}$.

Moduli space of interest: $\overline{M}_{D_{1,6}}$, defined as the Zariski closure in $\overline{M}^{KSBA}$ of $M_{D_{1,6}}$ which parametrizes $(S, \epsilon\sum_{i=1}^{3}(E_i + E_i'))$.

Main results

Theorem (S, 2016)

1) $\overline{M}_{D_{1,6}} \cong \overline{M}^{STP}/(\textit{finite group})$, *where* $\overline{M}^{STP}$ *is a toric variety which is a moduli space parametrizing stable toric pairs.*

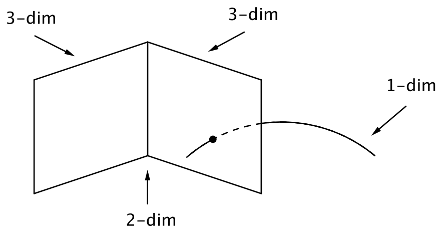
2) *Complete classification of the stable pairs parametrized by points in* $\overline{M}_{D_{1,6}}$. *In particular:*

- *Isolated singularities:* A_1, $\frac{(1,1)}{4}$.
- *Smooth irreducible components degenerations:* $D_{1,6}$*-polarized Enriques surfaces,* $\mathbb{P}^1 \times \mathbb{P}^1$*, genus* 1 *fibrations, weak del Pezzo surfaces of degree 1, del Pezzo surfaces of degree* 2 *and* 4*.*

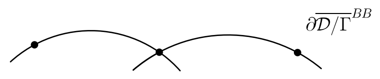
Main results

Theorem (S, 2016, continued)

3) $\partial \overline{M}_{D_{1,6}} = D_1 \cup D_2 \cup C$.



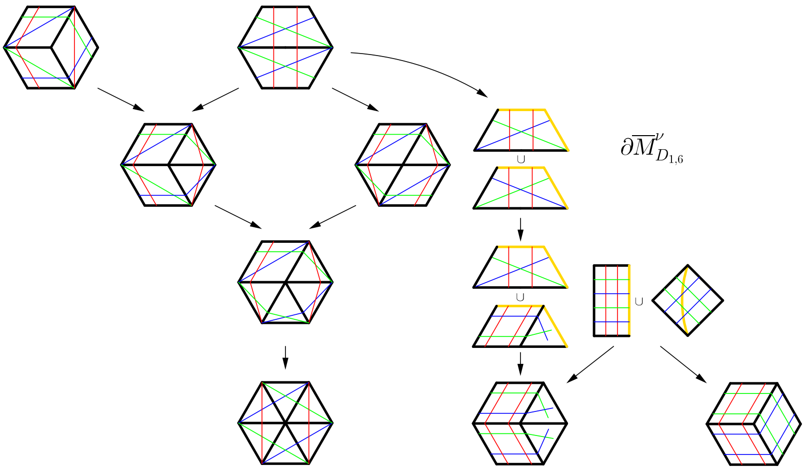
4) $\exists\ \overline{M}_{D_{1,6}} \xrightarrow{birational} \overline{\mathcal{D}/\Gamma}^{BB}$.



Thank you for your attention!

For the figures I used the software GeoGebra.

$\partial\overline{M}_{D_{1,6}}$ and modular interpretation of its points



$\partial\overline{M}_{D_{1,6}}$ and $\partial\overline{\mathcal{D}/\Gamma}^{BB}$ compared